Engineering Notes

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Critical Bending Moment of Double-Slit Tubing

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Nomenclature

A,B,C,D,F,H	= integration constants, m
c	= beam curvature, m^{-1}
d	= tube diameter, m
E	= Young's modulus, Pa
M	= beam bending moment, Nm
N	= see Eq. (4), m
r	= tube radius, m
t	= tube wall thickness, m
u	= longitudinal displacement, m
v	= tangential displacement, m
w	= radial displacement, m
Z	= section modulus, m^3
θ	= angular coordinate
ν	= Poisson's ratio

Introduction

THE present analysis is devoted to an investigation of the response to bending of double-slit, thin-walled circular tubing, in use for deployable spacecraft booms, and commonly referred to as BI-STEM tubing by space engineers. The analysis represents an extension of the theory used in previous papers[1,2] where the ovalling and overlapping effects of slit, thin-walled circular tubes subjected to bending are discussed. For purposes of the analysis, end effects are eliminated by assuming that the tube is long. Furthermore, it is assumed that the originally straight tube is bent into a large toroid section of uniform curvature c by a uniform bending moment M. Any influence of a variable curvature is thus also eliminated. The cross section of the double-slit circular tube before deformation is shown in Fig. 1.

Prior to instability by the effects discussed in the paper, instability resulting from local buckling may occur. Its effect, however, has been disregarded in the present analysis.

Basic Theory

This Note uses the approach originally introduced by Brazier,[3] among several approaches[3-5] available.

The tubes under consideration are thin-walled, i.e., $t \ll r$, such that the basic formulations for the theory of elastic shells apply. Let the derivation of the expression for the tangential displacement v and the radial displacement w of an arbitrary point P of the tube wall be based upon the assumption that the deformation is inextensional, i.e., that

$$w = v' \tag{1}$$

where

$$v' = \frac{dv}{d\theta} \tag{2}$$

For the present problem the elastic strain energy can be composed of two separate parts. One part is due to the normal stress and the axial displacement u associated with the bending of the tubular beam into a section of a toriod of curvature c. The second part is due to bending stress and the tangential and radial displacements v and w associated with the distortion of the orignally circular cross section.

In order for the strain energy to be a minimum, for a given fixed curvature c, the Euler-Langrange equation of the calculus of variations is applied, which results in a sixth-order differential equation

$$v^{vi} + 2v^{iv} + v'' = -N\sin 2\theta \tag{3}$$

with

$$N = 18(1-\nu^2)r^5c^2/t^2 \tag{4}$$

for simplicity.

The general solution of Eq. (3) is

$$v = A\cos\theta + B\sin\theta + C\theta\cos\theta + D\theta\sin\theta$$
$$+ F\theta + H + (N/36)\sin 2\theta \tag{5}$$

where A, B, C, D, F, and H are integration constants.

Cross-Sectional Change

At the slits only compression forces can be transmitted. For the loading case where the plane of bending is perpendicular

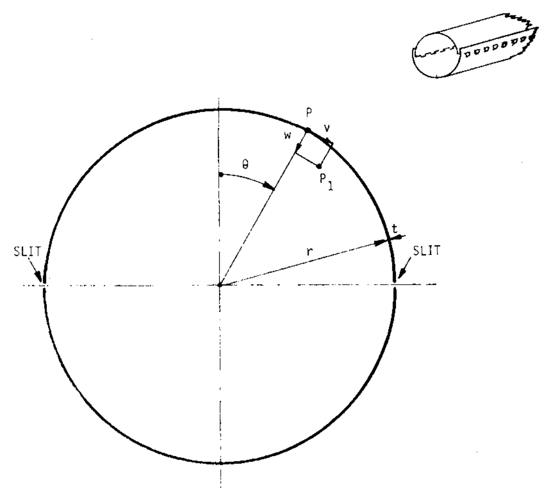

Fig. 1 Actual and idealized cross section of double-slit tubing.

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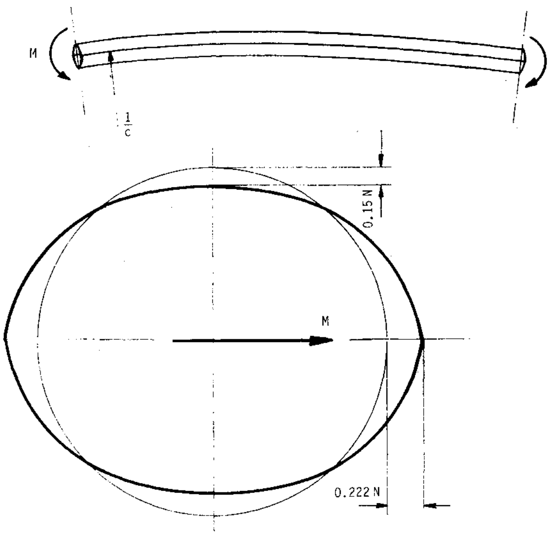

Fig. 2 Deformed cross section of double-slit tubing during bending.

to the plane containing the slits (Fig. 2), the natural boundary conditions supplied by the calculus of variations are

$$v''' + v' = 0, \qquad v^{iv} + v'' = 0$$

at $\theta \pm 90$ deg. Further boundary conditions include symmetry in radial displacement about $\theta = 0$ deg, no rigid body rotation, and no tangential displacment at $\theta = \pm 90$ deg. Consequently, $A = C = D = H = 0$, $B = \pi N/12$, and $F = -N/6$, such that the tangential displacement becomes

$$v = \frac{\pi}{12} N\sin\theta - \frac{N}{6}\theta + \frac{N}{36}\sin 2\theta \tag{6}$$

and the radial displacement

$$w = v' = \frac{\pi}{12} N\cos\theta - \frac{N}{6} + \frac{N}{18}\cos 2\theta \tag{7}$$

giving

$$w = \frac{3\pi - 4}{36} N = 0.15\,N \qquad \text{at} \quad \theta = 0 \text{ deg}$$

and

$$w = -(2/9)N = -0.222\,N \qquad \text{at} \quad \theta = \pm 90 \text{ deg}$$

See Fig. 2.

With the integration constants known, the strain energy can now be expressed in terms of the tube curvature c as sole variable.[1] By differentiating the strain energy with respect to c, the bending moment transmitted by the cross section of a tube is obtained as

$$M = E\pi r^3 t\left(c - \frac{9}{2}\frac{(1-\nu^2)r^4c^3}{t^2}\right) \tag{8}$$

Instability of the bending process occurs when $\partial M/\partial c = 0$, giving the critical bending moment

$$M_{\rm cr} = \frac{2\sqrt{2}\pi}{9\sqrt{3}}\frac{Ert^2}{\sqrt{1-\nu^2}} = 0.57\frac{Ert^2}{\sqrt{1-\nu^2}} \tag{9}$$

a result differing from Brazier[3] for a circumferentially closed tube by a factor of $1/\sqrt{3}$, attesting to the greater flexibility of the cross section of the double-slit tube.

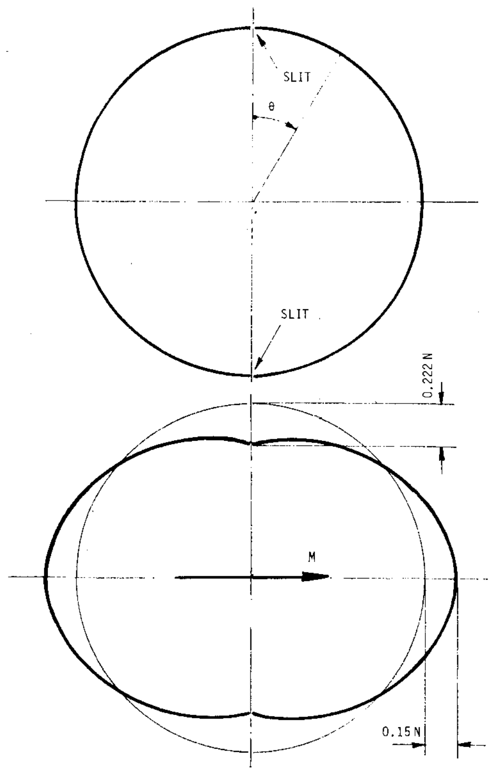

Fig. 3 Undeformed and deformed cross section of double-slit tubing during transverse bending.

Transverse Bending

The case where the plane of bending is coincident with the plane containing the slits (Fig. 3), also can be readily dealt with. Natural and imposed boundary conditions lead to integration contants $A = \pi N/12$, $B = C = D = 0$, $F = N/6$, $H = -\pi N/12$, such that the tangential displacement becomes

$$v = \frac{\pi N}{12}\cos\theta + \frac{N}{6}\left(\theta - \frac{\pi}{2}\right) + \frac{N}{36}\sin 2\theta \tag{10}$$

and the radial displacement

$$w = -\frac{\pi N}{12}\sin\theta + \frac{N}{6} + \frac{N}{18}\sin 2\theta \tag{11}$$

giving

$$w = \frac{2}{9}N = 0.222\,N \qquad \text{at} \quad \theta = 0 \text{ deg}$$

and

$$w = -\frac{3\pi - 4}{36}N = -0.15\,N \qquad \text{at} \quad \theta = 90 \text{ deg}$$

See also Fig. 3.

The bending moment for transverse bending can be shown to be

$$M = E\pi r^3 t\left(c - \frac{9}{2}\frac{(1-\nu^2)r^4c^3}{t^2}\right) \tag{12}$$

A comparison with Eq. (8) shows that the two are the same. The critical bending moment associated with transverse bending is consequently the same as given by Eq. (9).

Conclusion

The critical moment prediction [Eq. (9)] suffers not only from idealizations inherent in the theory, but also from the cross-sectional idealization (no overlap) used. BI-STEM tubes inevitably have some overlap in reality (Fig. 1) and are interlocked in some fashion, both circumstances providing added strength. Furthermore, there is experimental evidence[6] which also shows that Eq. (9) is conservative. The fact that the actual cross section is typically greater than the one used in the present theory can be taken into account by introducing the section modulus Z as basis, rather than the radius r. The section modulus for the cross section used to derive Eq. (9) is

$$Z = \pi r^2 t \tag{13}$$

such that Eq. (9), with $\nu = 0.3$, can be expressed as

$$M_{cr} = 0.38EZ\frac{t}{d} \tag{14}$$

where $d = 2r$ is the tube diameter. In this form, with the section modulus Z of the actual cross section, the equation is suitable for critical bending moment prediction for double-slit tubing with overlapping and interlocking.

References

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A Simple Estimation Procedure of Roll-Rate Derivatives for Finned Vehicles

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Introduction

SOME of the important aspects that warrant consideration in the prediction of the roll-rate derivatives (C_{lp} and $C_{l\delta}$) are 1) normal-load distribution over the fin planform, 2) choice of the proper fin-body interference factor, and 3) the cruciform configuration effects. Barrowmann et al.[1] have brought out a simple and logical approach for the estimation of C_{lp} and $C_{l\delta}$. However, their approach considers a uniform load, $CN_{\alpha f}$, over the fin in the subsonic regime which may lead to overprediction of the roll-rate derivatives. Even in the supersonic regime the use[1] of Busemann's higher-order theory for small incidence may lead to slight overprediction as studied by Oberkampf.[2] Further, many investigators[1-4] do not appear to have considered the cruciform effects, as brought out by Adams and Dugan.[5]

The present Note briefly outlines a more generalized formulation for computing $C_{l\delta}$ and C_{lp}, discusses the adaption of proper fin-body and fin-fin interference effects, and presents a comparison of the present calculations with experimental data which are found to be in good agreement. However, the present approach overpredicts C_{lp} at subsonic speeds in certain cases.

Analysis

Roll Moment Calculation

The roll moment coefficient for planar fin (suffix-) can be derived as

$$C_{l-} = \frac{2}{SD}\int\int_{A'} \Delta C_p \alpha(\xi)\xi \,dx\,d\xi \tag{1}$$

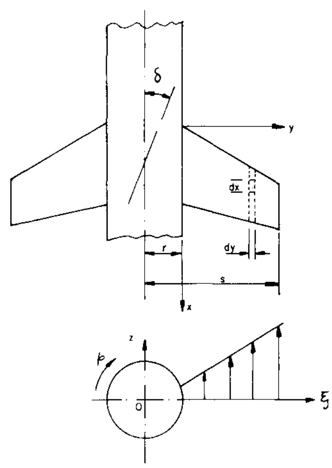

Fig. 1a Roll parameters.

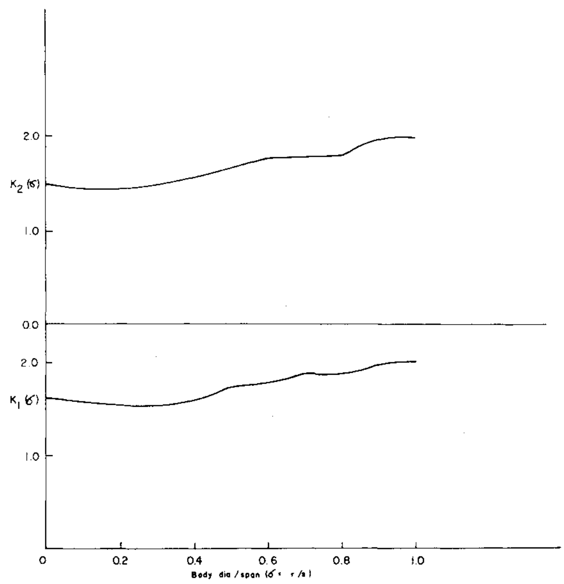

Fig. 1b Variation of factors K_l and K_2.

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